

Concept Review

- Eigenvalues and Eigenvectors.
- Trace of a Matrix and its Properties.
- Similar Matrices.
- Spectral Mapping Theorem.
- Minimal Polynomial.

Skills

- Find the eigenvalues of a matrix.
- Find bases for the eigenspaces of a matrix.
- Prove some theorems.

1. Find the eigenvalues and bases for the eigenspaces of the matrix

$$A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

- 2. Show that the characteristic equation of 2×2 matrix A can be expressed as $\lambda 2 tr(A)\lambda + \det(A) = 0.$
- 3. Prove that if λ is an eigenvalue of A, x is a corresponding eigenvector and s is a scalar, then λs is an eigenvalue of A sI and x is a corresponding eigenvector.
- 4. Find det(A), tr(A) given that A has characteristic polynomial

$$p(\lambda) = \lambda^3 - 2\lambda^2 + \lambda + 5$$

- 5. Prove that similar matrices have the same trace.
- 6. Show that if A and B are similar invertible matrices, then A^{-1} and B^{-1} are similar.
- 7. Give a definition for the minimal polynomial of a matrix A.